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THEORETICAL CONSIDERATIONS ON THE DRAG FORCE FOR A LIFTING WING WITH FLUID EMISSION FROM THE TRAILING EDGE

BY

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Abstract. The work is proposing a theoretical method of drag forces establish for a lifting wing. The method considers the analysis of real fluid flow in the back of the wing. The study is made to sufficiently big distances that the movement could be considered with potential character. It is followed the jet's influence on the back flow. It is observed the circulation when is taken in consideration the fluid emission from the trailing edge. It is theoretically estimated the speed potential and the scalar components in case of a bivariated movement. It is established a theoretical relation for the drag coefficient when the fluid emission from the trailing edge is controlled.

Keywords: wing; fluid emission; circulation; potential movement; drag force.

1. General Considerations

In specialty literature, it is shown that a considerable part of the drag force is determined by the resistance bounded by the energy dissipated in the layer formed in the back of aerodynamic profile, named turbulent backwater.

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So, the drag force F_x, written in total differential form of impulse in ratio with x component in which direction is made the movement, for two sections determined by the coordinates $x = x_1$ and $x = x_2$, situated in the back of the profile, but enough away by the body, and in the front of the profile too, it is determined with the relation for total resistance (Oroveanu 1976; Reigels 1958).

$$F_x = \left[\iint\limits_{x=x_2} - \iint\limits_{x=x_1} \right] \left[p + \rho (u_\infty - u)^2 \right] dy \cdot dz \tag{1}$$

in which: p – fluid current pressure; ρ – density of fluid being in movement around the body; u_∞ – fluid current speed in the infinite section; u – speed component in Ox direction, direction that coincides with general movement direction. This speed is considerate in the next vicinity of profile surface, into the limit layer.

Because the aerodynamic trace is thin, it could be negligee the integral from the plane $x = x_1$, the integral on the section surface. In this case, applying the Bernoulli' rule, it can be determined the resistance force with relation:

$$F_x = \left[\iint\limits_{x=x_2} - \iint\limits_{x=x_1} \right] \left[p + \rho \cdot u_\infty^2 + \rho \cdot u_\infty \cdot u + \frac{\rho}{2} (u^2 + v^2 + w^2) \right] dy \cdot dz \,. \tag{2}$$

The difference of the integrals for the value of $p + \rho \cdot u_\infty^2$ is constant, and becomes null. The difference of integrals $\rho \cdot u_\infty \cdot u$ is eliminated because the fluid flux $\iint \rho \cdot u \cdot dy \cdot dz$, which crosses the two sections taken in consideration and the back of the profile, must be equal, in conformity with the continuity equation, after this direction.

By moving the plane $x = x_2$ to a distance enough big by the body in the front of those, it is observed that the vectorial speed *v* from the internal limit layer of thickness δ is relatively small, which permitted the inconsideration of term $\rho \left[u^2 + v^2 + w^2 \right]$ like being a small value of second order. The values *u*, *v* and *w* are scalar components of speed v, from limit layer.

If it is taken in consideration the hypothesis that *u* is smaller than components *v* and *w* (in front vicinity of braking out point) and out of backwater, then the relation that permitted the determination of drag force becomes:

$$F_x = \frac{\rho}{2}\iint\left[v^2 + w^2\right]dy \cdot dz\,. \tag{3}$$

The resistance so calculated for an aerodynamic wing could be written in function of speed's circulation Γ. This circulation determines the profile's lift capacity. Remarkable is the fact that, at big distances from the profile, the speed depends only by the coordinates y and z, so that their scalar components are:

$$v = v(y,z) \quad \text{and} \quad w = w(y,z)\ . \tag{4}$$

So, in this zone the fluid movement can be assimilated with a plane movement, which permits introduction of current function $\psi = \psi(y,z)$, which can establish analytical relations for the resolving of those two components of established speeds in the relation (4):

$$v = -\frac{\partial\psi(y,z)}{\partial z}\cdots\text{and}\cdots w = \frac{\partial\psi(y,z)}{\partial y}\,. \tag{5}$$

Considering the relation (3), for the drag force it could be established the next analytical formula:

$$F_x = \frac{\rho}{2}\iint\left[\left(\frac{\partial\psi(y,z)}{\partial z}\right)^2 + \left(\frac{\partial\psi(y,z)}{\partial y}\right)^2\right]dy \cdot dz\,. \tag{6}$$

Relation, which, for being true, must be, created the hypothesis that, to the chosen referee system, the y coordinate goes to ∞ (Pop, 1983).

Because the fluid's general flowing in the external backwater has potential character, it exist so the function $\phi = \phi(y,z)$, named potential speed function, which satisfied the Laplace equation:

$$\frac{\partial^2\varphi(y,z)}{\partial y^2} + \frac{\partial^2\varphi(y,z)}{\partial z^2} = 0\,. \tag{7}$$

For the application of the relation (6) of the bivariated Green's formula, it is obtained for the drag force the next expression:

$$F_x = -\frac{\rho}{2}\int\psi(y,z)\cdot\frac{\partial\psi(y,z)}{\partial n}dl\,. \tag{8}$$

The integral being effectuated in the length of body's contour and $\partial / \partial n$ being the gradient in the length of external normal to contour.

In the point from infinite $\psi(y,z) = 0$ and, in consequence, the integration must be effectuated on a transversal contour of the backwater.

In consequence of this hypothesis, it is obtained for the drag force the next relation:

$$F_x = \frac{\rho}{2} \int \psi(y,z) \left[\left(\frac{\partial \psi(y,z)}{\partial y} \right)_2 - \left(\frac{\partial \psi(y,z)}{\partial y} \right)_1 \right] dz \,. \qquad (9)$$

In this case, the integration is made on elementary length dz of the backwater and the difference between the current function variations in the two sections considered represent the loop, which the derivate $\partial \psi(y,z) / \partial y$ suffers on traversing the backwater.

If it is considered the fact that $\partial \psi(y,z) / \partial y = w = \partial \phi(y,z) / \partial z$ then it is obtained:

$$\left[\frac{\partial \psi(y,z)}{\partial y} \right]_2 - \left[\frac{\partial \psi(y,z)}{\partial y} \right]_1 = \left[\frac{\partial \varphi(y,z)}{\partial z} \right]_2 - \left[\frac{\partial \varphi(y,z)}{\partial z} \right]_1 = \frac{d\Gamma}{dz}, \qquad (10)$$

that could permit the obtaining of the relation for the drag force

$$F_x = \frac{\rho}{2} \int \psi(y,z) \frac{d\Gamma}{dz} dz \,. \qquad (11)$$

In conformity with the potential movement theory, it is established for the current function $\psi(y,z)$ the expression:

$$\psi(y,z) = -\frac{1}{2\pi} \int \left[\left(\frac{\partial \psi(y,z)}{\partial n} \right)_2 - \left(\frac{\partial \psi(y,z)}{\partial n} \right)_1 \right] \ln r \cdot dl \,. \qquad (12)$$

The integration being effectuated on a plane contour; *r* is the distance to a point from this plane where it is looked to determine the current function's value $\psi(y,z)$. The measure between the parentheses represents the loop of current function's derivate $\psi(y,z)$ in the length of normal on the plan contour, where is made the integration.

In case of relation (12), the integration may be realized only in the length of a segment of axe Oz so that in this situation the value of current function becomes the monovariable $\psi = \psi(z)$ and has the next expression:

$$2\pi \cdot \psi(z) = \int \left[\left(\frac{\partial \psi(z)}{\partial y} \right)_2 - \left(\frac{\partial \psi(y)}{\partial y} \right)_1 \right] \cdot \ln\left|z - z^{'}\right| dz^{'} = -\int \frac{d\Gamma(z^{'})}{dz^{'}} \cdot \ln\left|z - z^{'}\right| \cdot dz \cdot dz^{'} \quad (13)$$

Introducing the relation (13) in drag force's expression (11), this becomes:

$$F_x = \frac{\rho}{4\pi} \iint \frac{d\Gamma(z^{'})}{dz^{'}} \cdot \frac{d\Gamma(z)}{dz} \cdot \ln\left|z - z^{'}\right| \cdot dz \cdot dz^{'} . \quad (14)$$

Prandtl gives the circulation's distribution law by developing a series of a trigonometric function:

$$\Gamma = -2 \cdot u_\infty \cdot l \sum_{n=1}^{\infty} A_n \cdot \sin(n\theta) . \quad (15)$$

2. Conclusions

Once determined the friction force with the relation (14) it could be established the expression of friction coefficient C_x, coefficient that determines the advancing resistance for the aerodynamic profile:

$$C_x = \pi \cdot \lambda \sum_{n=1}^{\infty} n \cdot A_n^2 . \quad (16)$$

Or considering the expression (14), the relation of friction coefficient becomes:

$$C_x = \frac{1}{2\pi u_\infty{}^2 l_x l_z} \iint \frac{d\Gamma(z)}{dz} \cdot \frac{d\Gamma(z')}{dz'} \ln\left|z - z'\right| dz dz' . \quad (17)$$

It could be observed that in aerodynamic profile theory, the most important effect of limit layer's back out on the up side of the profile is consisting in the increase of advancing resistance.

The limit layer's back out is made, in generally, in a point situated in the next of leading edge. It is imposed the translation of this point in the next of trailing edge, or this thing is realized only using control methods for the limit layer.

One of those methods consists in launching an air jet, with big speed, between a slots placed in trailing edge.

In this case, besides the maintaining a positive speed gradient in length of upside of the profile, the apparition of a horizontal component of a reactive force into the jet, component which is collinear with the advancing resistance but with inverse sense, determines the considerable decreasing of advancing resistance force and implicit of coefficient C_x.

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CONSIDERAȚII TEORETICE ASUPRA FORȚEI DE REZISTENȚĂ LA ÎNAINTARE ÎN CAZUL ARIPII PORTANTE CU EMISIE DE FLUID PRIN BORDUL DE FUGĂ

(Rezumat)

Lucrarea propune o metodă teoretică pentru determinarea forței de rezistență la înaintare pentru o aripă portantă. Metoda consideră analiza mișcării fluidului real în spatele aripii la o distantă suficient de mare astfel încât mișcarea să aibă un caracter potențial. Este estimată circulația în cazul aripii cu emisie de fluid prin bordul de fugă.

Este estimat teoretic potențialul vitezei și componentele scalare pentru mișcarea bidimensională. Este stabilită o relație teoretică de calcul pentru coeficientul de rezistență la înaintare pentru aripa cu jet controlat în bordul de fugă.